P-values and statistical tests
7. Statistical power

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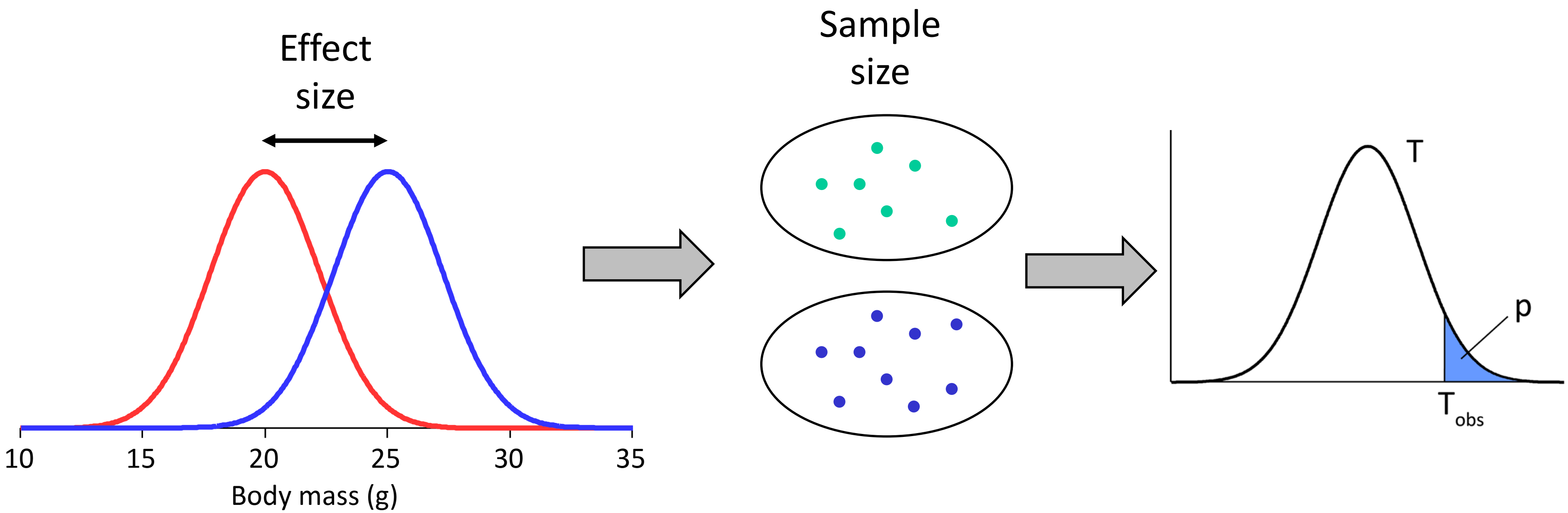
Hand-outs available at http://is.gd/statlec

Statistical power: what is it about?



Two populations (alternative hypothesis)

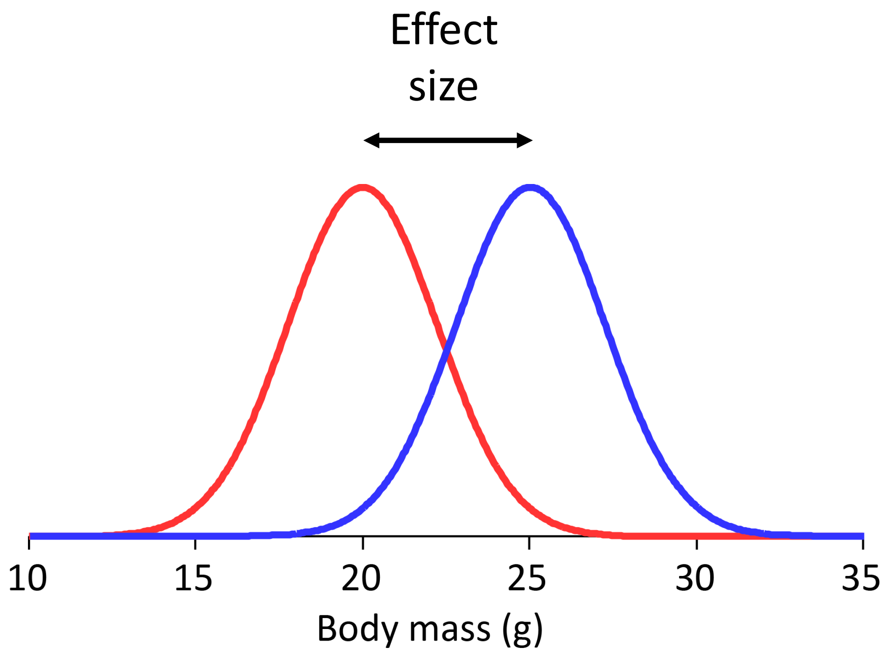
Two samples

Statistical significance

How does our ability to call a change “significant” depend on the effect size and the sample size?

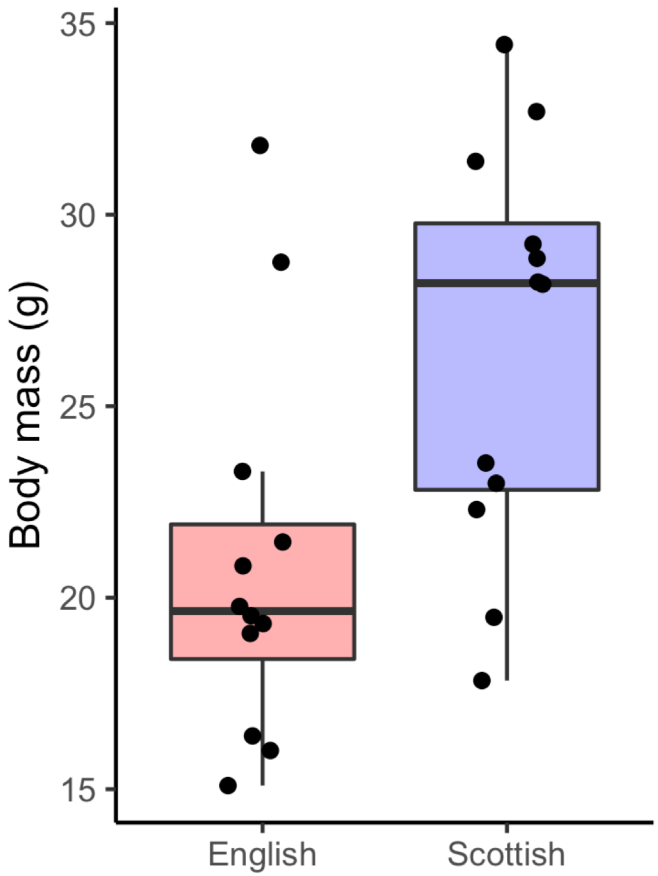
Effect size

Effect size describes the alternative hypothesis



Effect size for two sample means

$d = 1.1$



$$d = \frac{M_1 - M_2}{SD}$$

Cohen's d

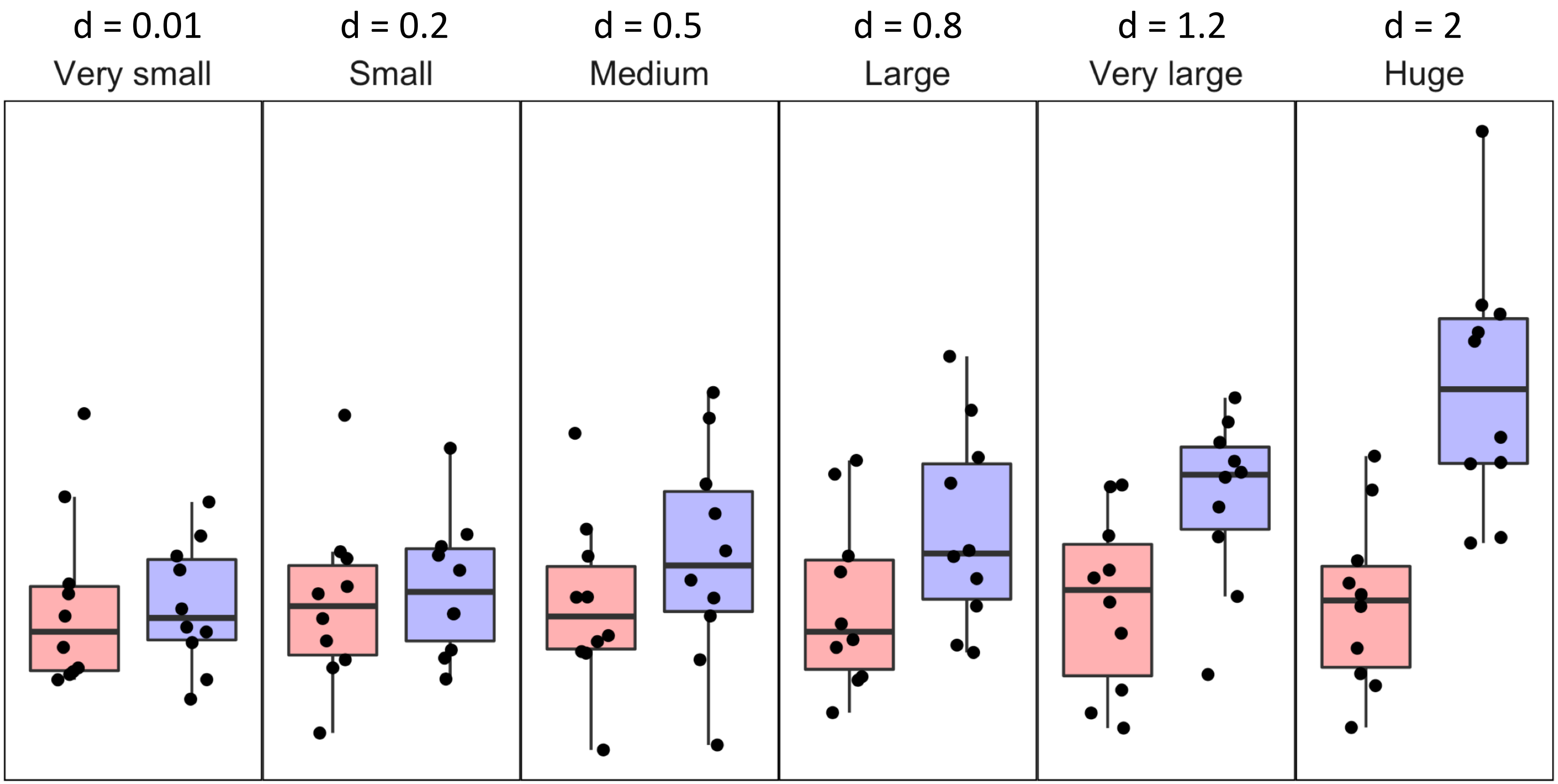
M – mean
SD – standard deviation

$$SD = \sqrt{\frac{(n_1 - 1)SD_1^2 + (n_2 - 1)SD_2^2}{n_1 + n_2 + 2}}$$

$$t = \frac{M_1 - M_2}{SE}$$

$$d = t\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$$

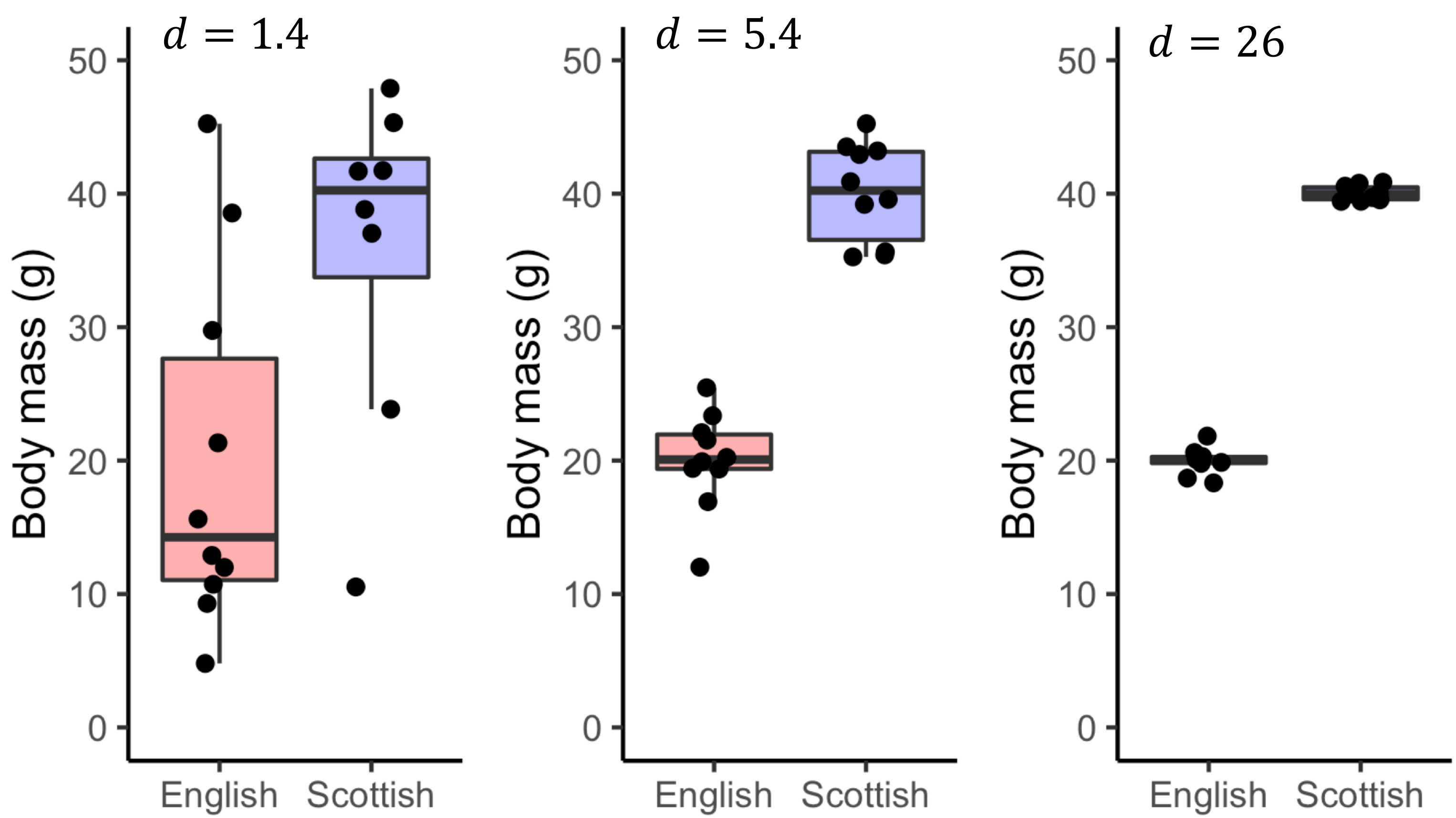
Effect size for two sample means



Cohen, J. (1988). *Statistical power analysis for the behavioral sciences*

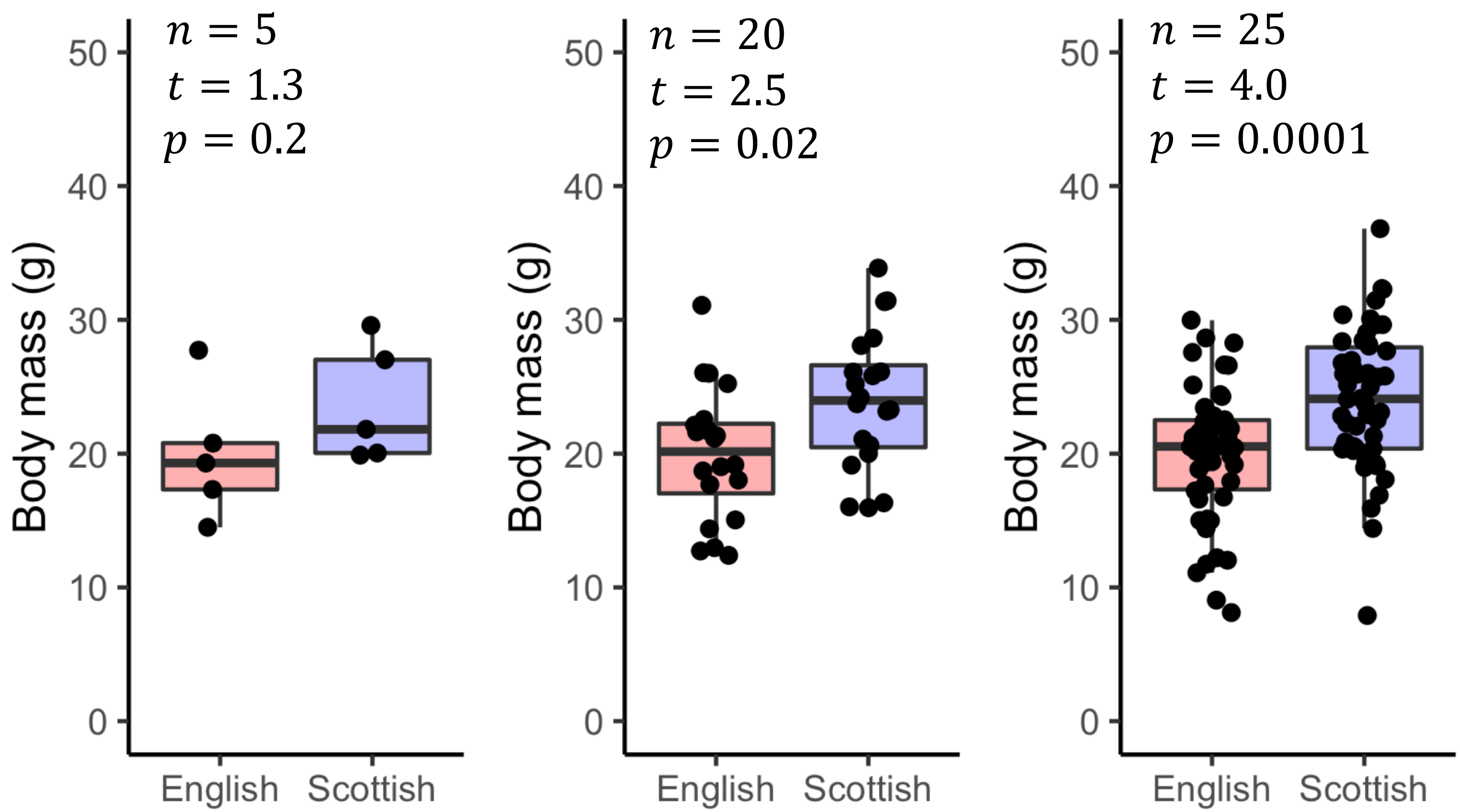
Effect size depends on the standard deviation

Fold change = 2
Difference between means = 20 g



Effect size does not depend on the sample size

Effect size = 0.8



Comparing two samples

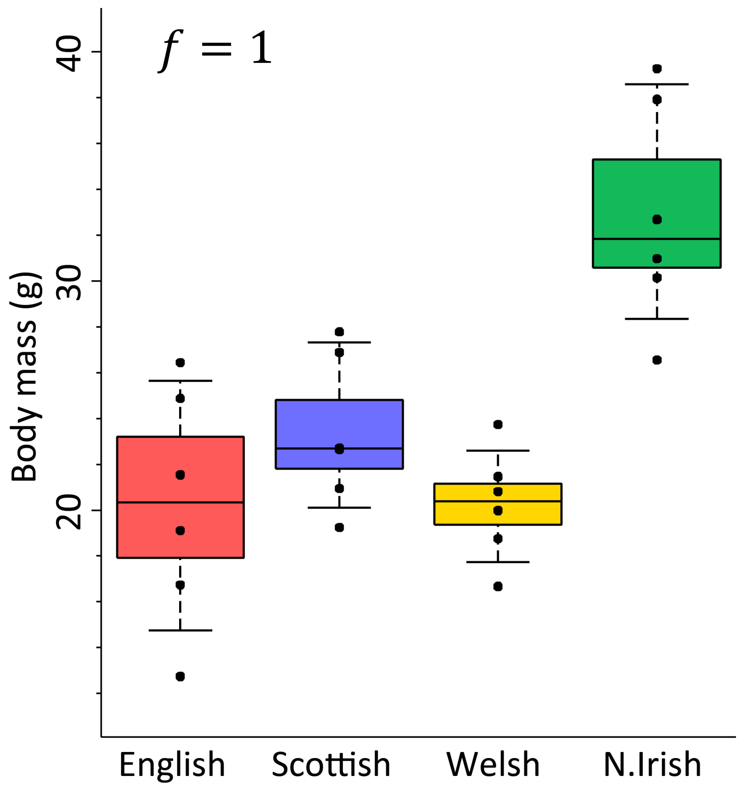
Statistic	Formula	Description
Difference	$\Delta M = M_1 - M_2$	Absolute difference between sample means
Ratio	$r = \frac{M_1}{M_2}$	Often used as logarithm
Cohen's d	$d = \frac{M_1 - M_2}{SD}$	Effect size; takes spread in data into account
t-statistic	$t = \frac{M_1 - M_2}{SE}$	Directly relates to statistical significance; takes spread of data and sample size into account

M – mean
SD – standard deviation
SE – standard error

Effect size describes the alternative hypothesis

Effect size is not related to statistical significance

Effect size in ANOVA



Test statistic

$$F = \frac{MS_B}{MS_W}$$

H_0: $MS_B = MS_W$

H_1: $MS_B = MS_W + nMS_A$

Added variance

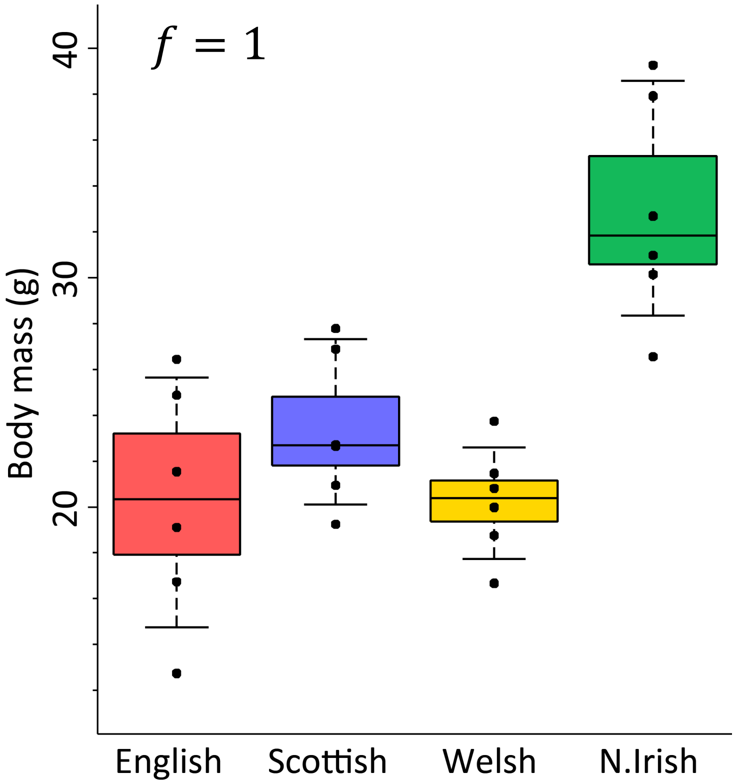
$$f^2 = \frac{MS_A}{MS_W}$$

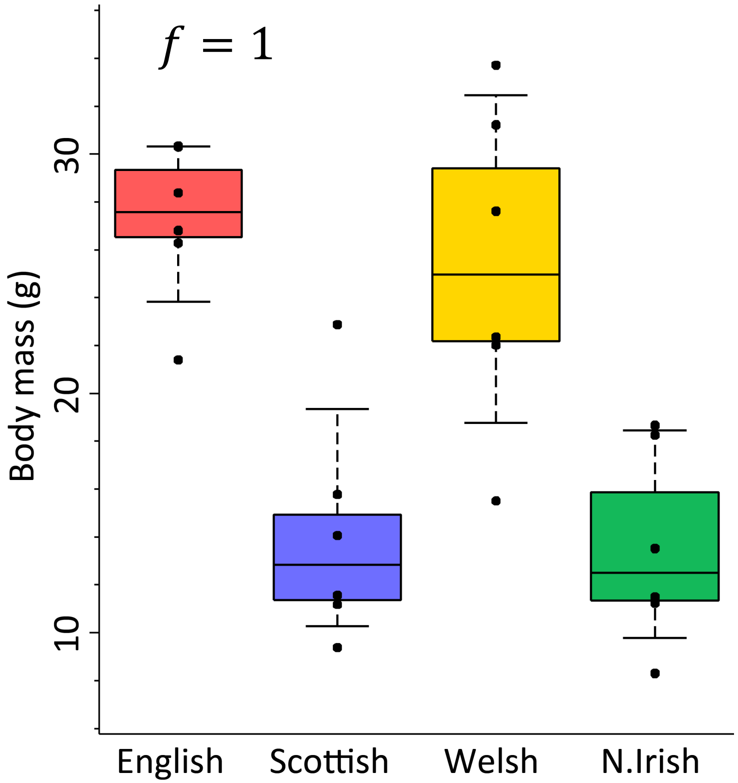
Cohen's f

$$f^2 = \frac{F-1}{n}$$

For the purpose of this calculation we only consider groups of equal sizes, n

Effect size in ANOVA





Effect size

Data	Statistical test	Effect size	Formula
Two sets, size n_1 and n_2	t-test	Cohen's d	$d = t\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$
k groups of n points each	ANOVA	Cohen's f	$f = \sqrt{\frac{F-1}{n}}$
2×2 contingency table	Fisher's exact	Odds ratio	$\omega = \frac{q_B/p_B}{q_A/p_A}$
Paired data $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$	Significance of correlation	Pearson's r	$r = \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{x_i - M_x}{SD_x}\right)\left(\frac{y_i - M_y}{SD_y}\right)$

Statistical power
t-test

Statistical testing

Statistical model

Null hypothesis

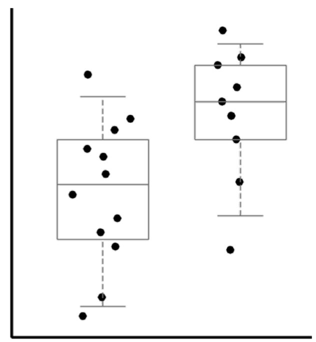
H_0: no effect

All other assumptions

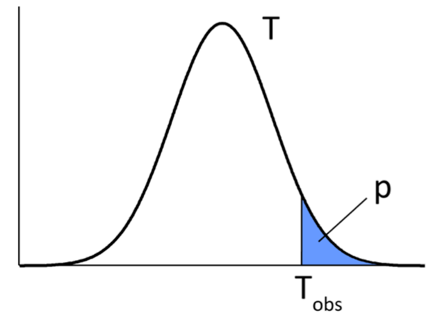
Significance level

$\alpha = 0.05$

Data



Statistical test against H_0



p-value: probability that the observed effect is random

$p < \alpha$

Reject H_0

(at your own risk)

Effect is real

$p \geq \alpha$

Accept H_0 (!!!)

This table

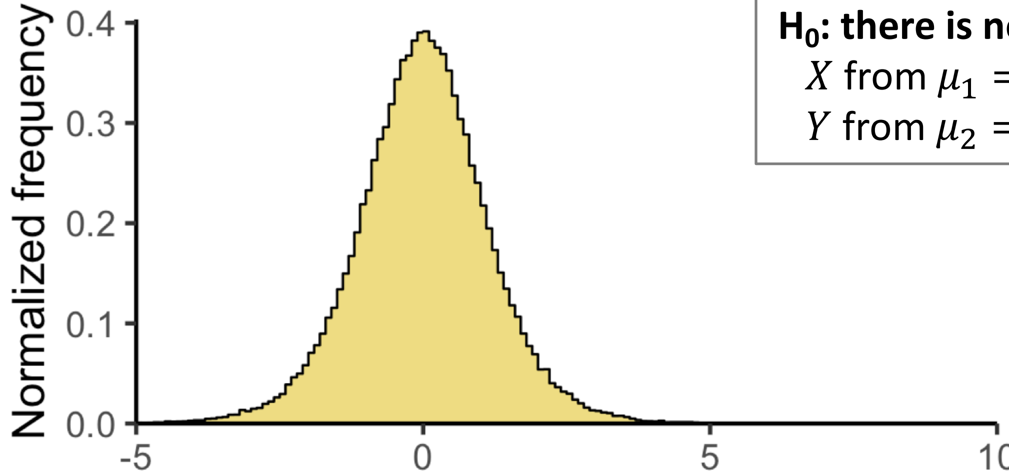
	H_0 is true	H_0 is false	
H_0 rejected	**type I error (α)** false positive	**correct decision** true positive	Positive
H_0 accepted	**correct decision** true negative	**type II error (β)** false negative	Negative
	No effect	Effect	

Gedankenexperiment

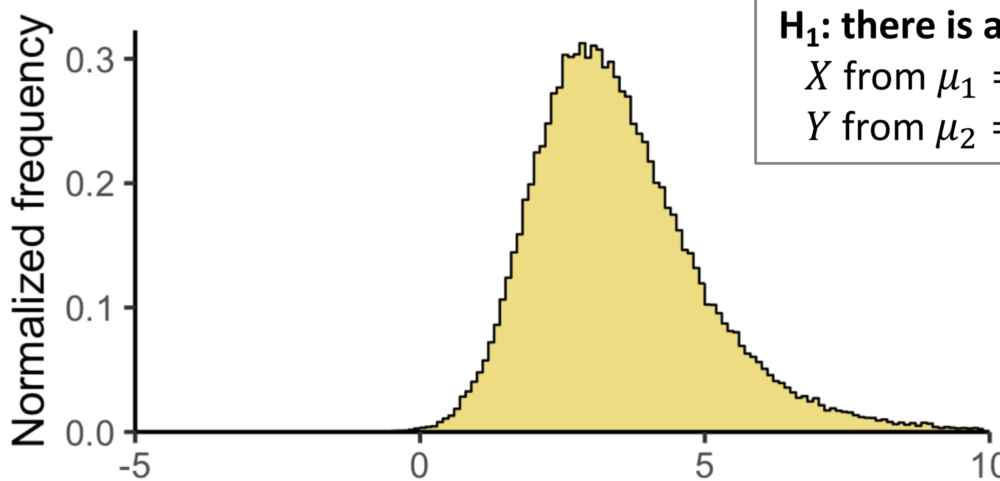
Draw 100,000 pairs of samples (X, Y) of size $n = 5$

Find $t = (M_1 - M_2)/SE$ for each pair

Build sampling distribution of t

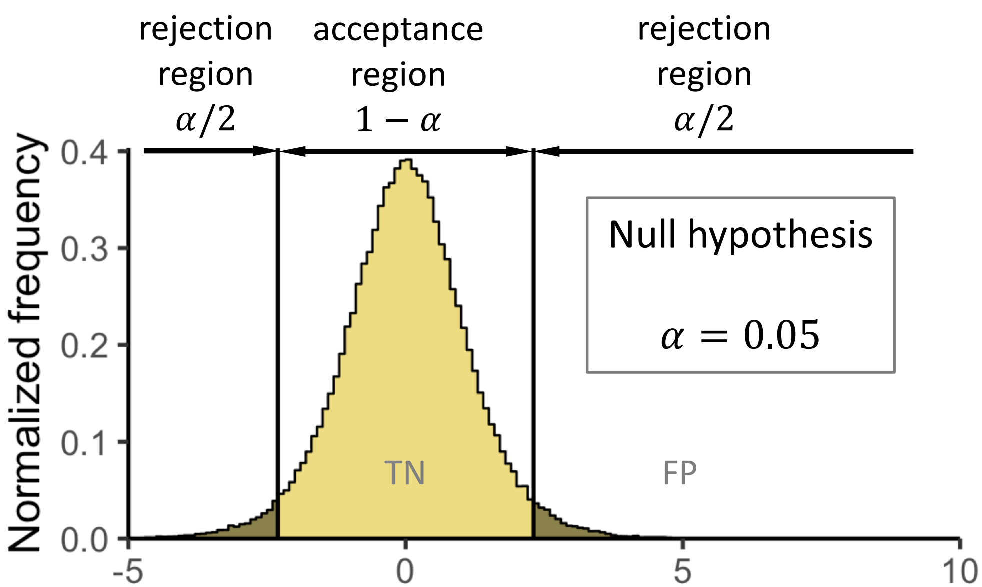


H_0: there is no effect
X from $\mu_1 = 20$ g
Y from $\mu_2 = 20$ g

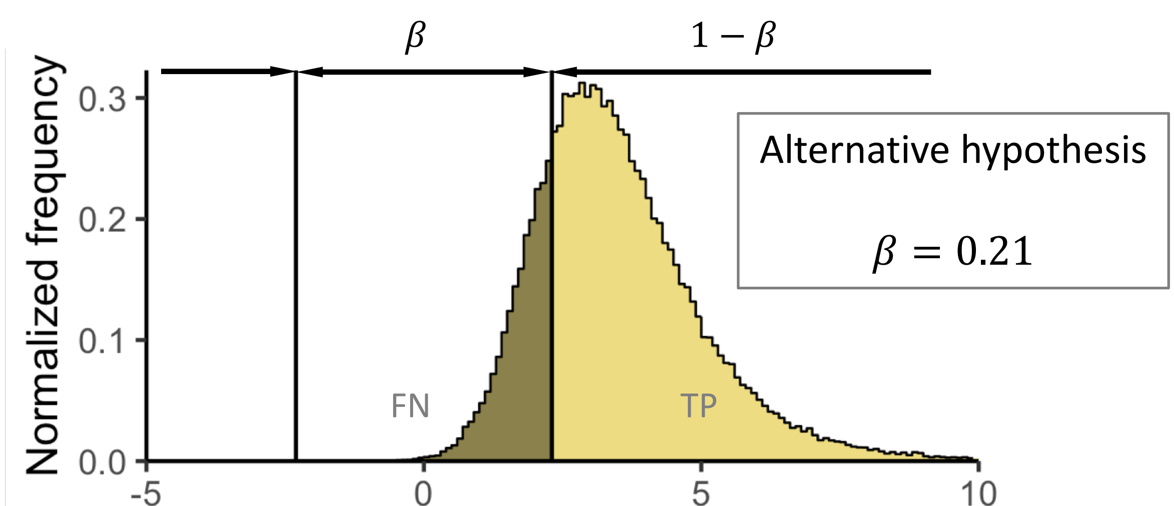


H_1: there is an effect
X from $\mu_1 = 20$ g
Y from $\mu_2 = 30$ g

One alternative hypothesis



	H_0 true	H_0 false
reject	FP $\boldsymbol{\alpha}$	TP
accept	TN	FN $\boldsymbol{\beta}$



Power of the test

$$P = 1 - \beta$$

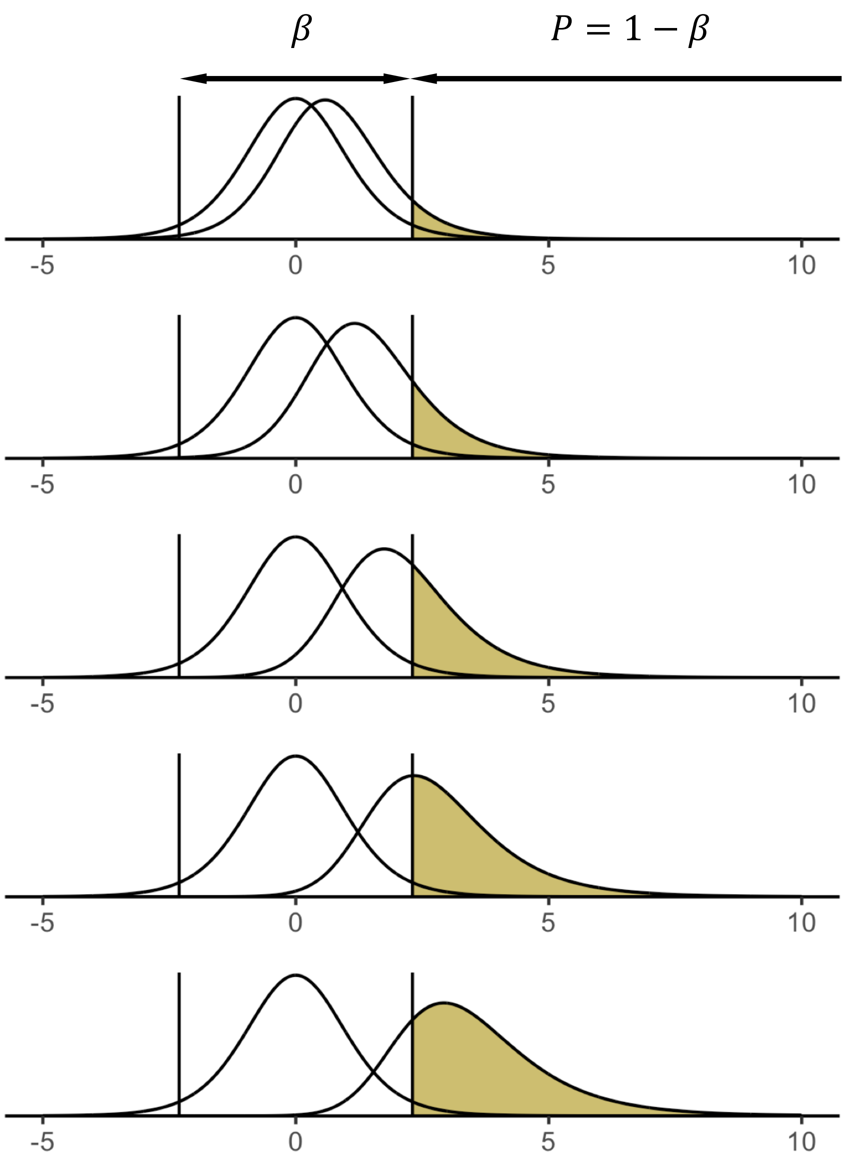
Probability that we correctly reject H_0

Statistical power

The probability of correctly rejecting the null hypothesis

The probability of detecting an effect which is really there

Multiple alternative hypotheses



$\mu_1 = 22$ g
$d = 0.4$
$P = 0.08$

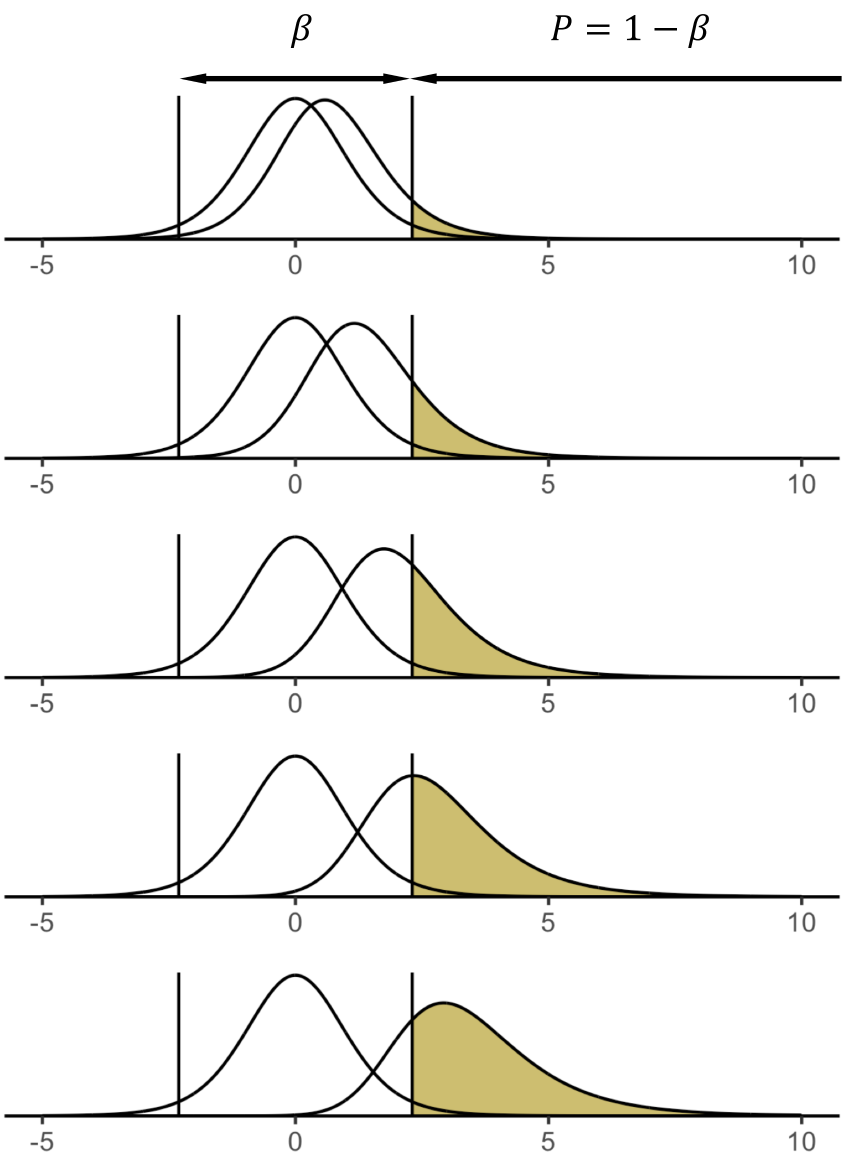
$\mu_1 = 24$ g
$d = 0.8$
$P = 0.20$

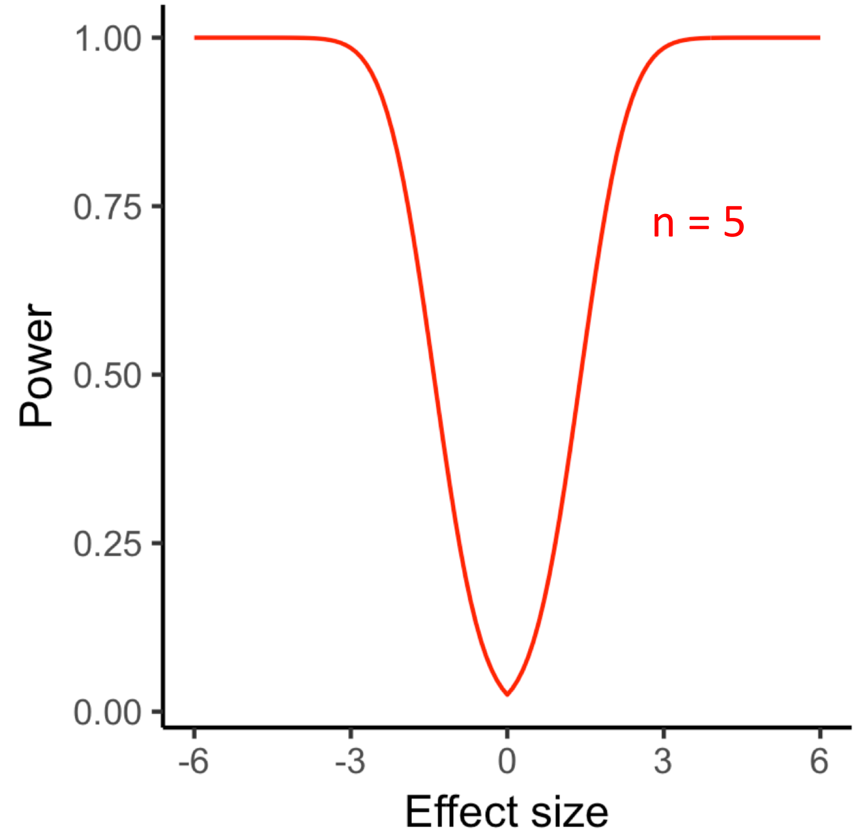
$\mu_1 = 26$ g
$d = 1.2$
$P = 0.39$

$\mu_1 = 28$ g
$d = 1.6$
$P = 0.60$

$\mu_1 = 30$ g
$d = 2.0$
$P = 0.79$

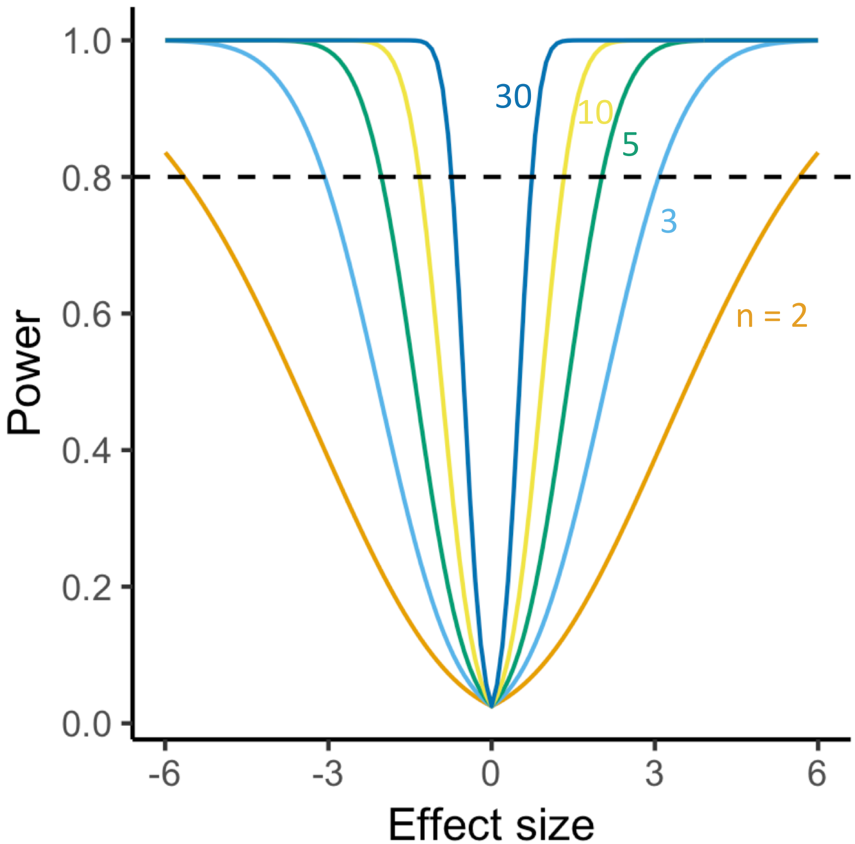
Power curve

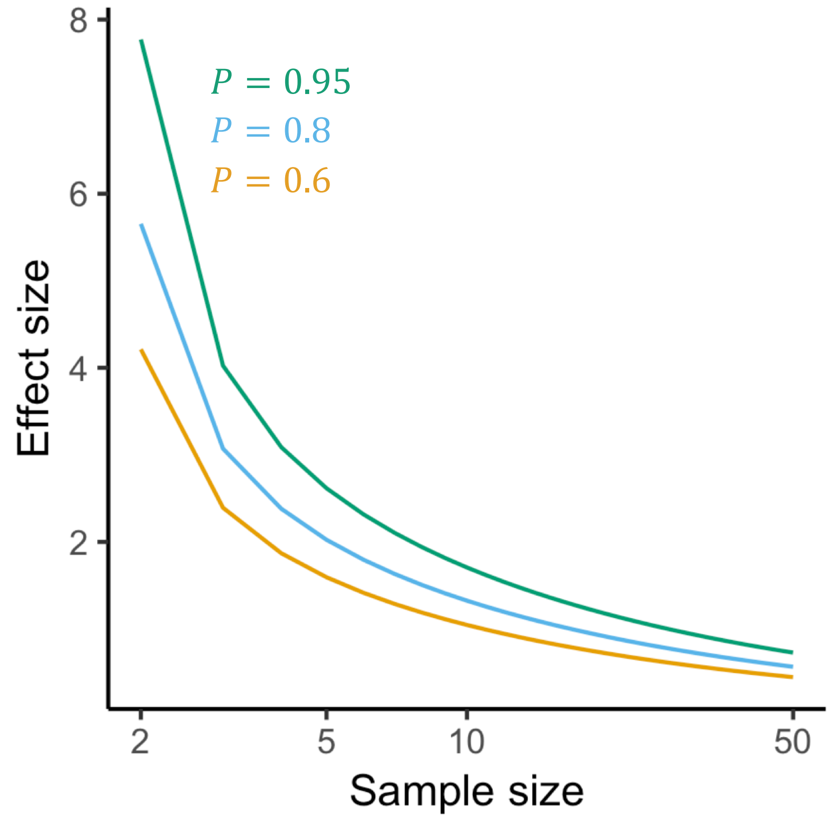




Power: probability of detecting an effect when there is an effect

Power curves





$$d = \frac{M_1 - M_2}{SD}$$

How to do it in R?

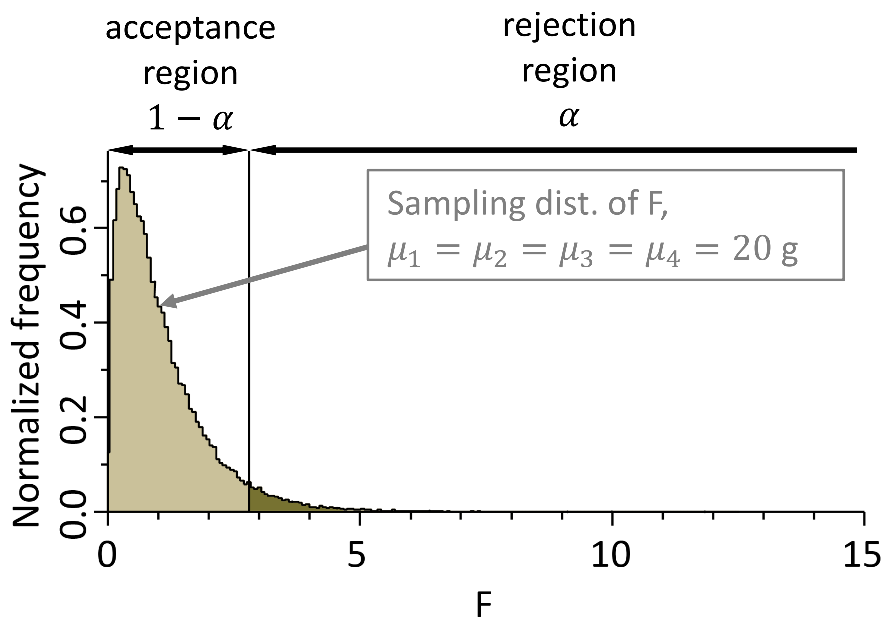
```
# Find sample size required to detect the effect size d = 1
> power.t.test(delta=1, sig.level=0.05, power=0.8, type="two.sample",
alternative="two.sided")

     Two-sample t test power calculation

              n = 16.71477
          delta = 1
             sd = 1
      sig.level = 0.05
          power = 0.8
    alternative = two.sided

> power.t.test(delta=1, sig.level=0.05, power=0.95, type="two.sample",
alternative="two.sided")

     Two-sample t test power calculation

              n = 26.98922
          delta = 1
             sd = 1
      sig.level = 0.05
          power = 0.95
    alternative = two.sided
```

Statistical power
ANOVA

One alternative hypothesis

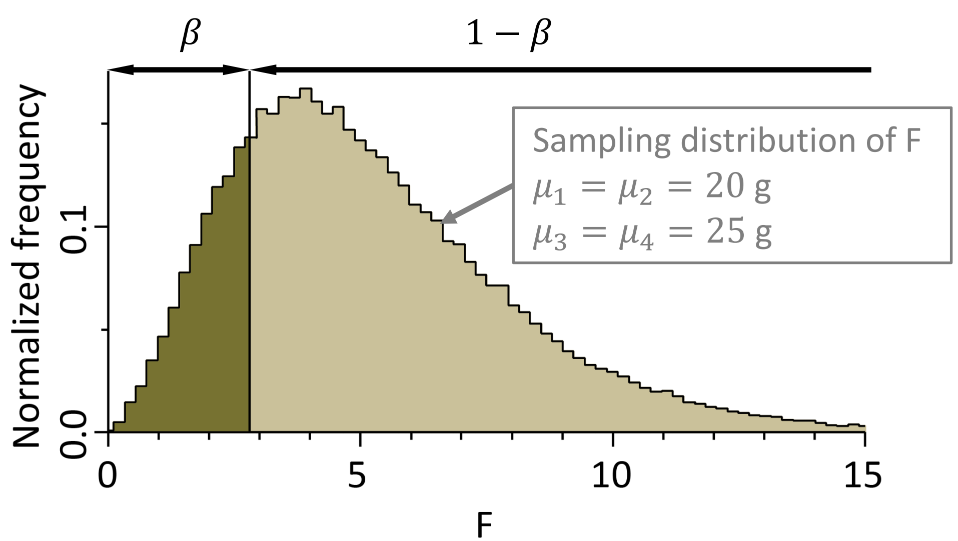
Null hypothesis

$\alpha = 0.05$

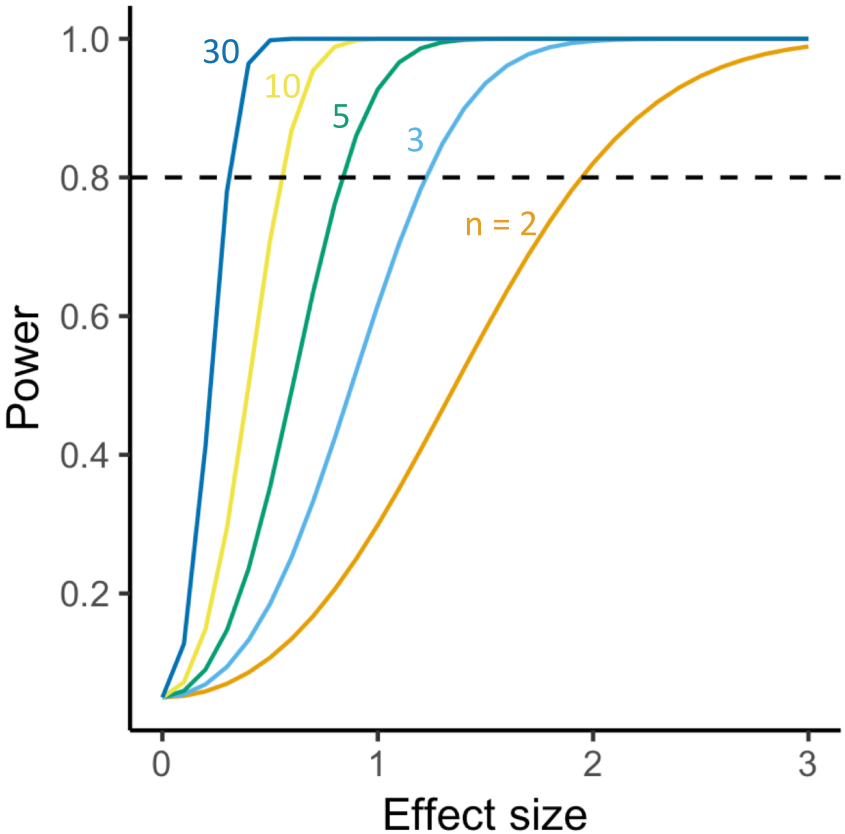


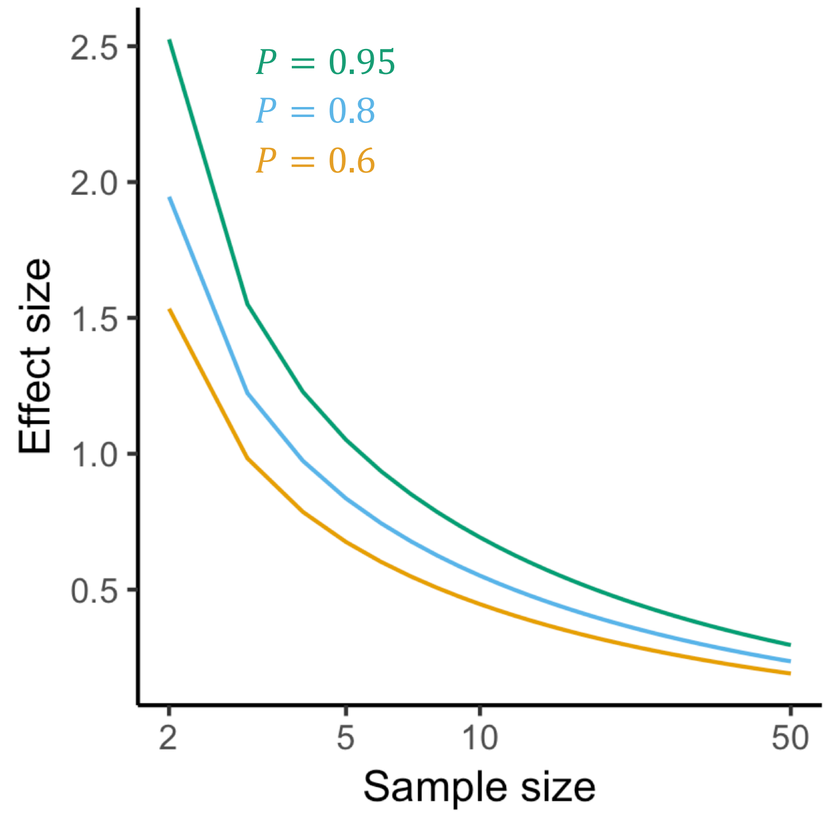
Alternative hypothesis

$\beta = 0.20$



Power curves





How to do it in R?

```
> library(pwr)

# Find sample size required to detect a “large” effect size f = 0.4
> pwr.anova.test(k=4, f=0.4, sig.level=0.05, power=0.8)

     Balanced one-way analysis of variance power calculation

              k = 4
              n = 18.04262
              f = 0.4
      sig.level = 0.05
          power = 0.8

NOTE: n is number in each group
```

Statistical power

Fisher's test

Power test for proportion

	Dead	Alive	Total
Drug A	68	12	80
Drug B	70	30	100
Total	138	42	180

Proportions in rows

	Dead	Alive	Total
Drug A	0.85	0.15	1
Drug B	0.70	0.30	1

```
# Find sample size required to detect
observed proportions
> power.prop.test(p1=0.85, p2=0.70,
power=0.8)

     Two-sample comparison of
proportions power calculation

              n = 120.4719
             p1 = 0.85
             p2 = 0.7
      sig.level = 0.05
          power = 0.8
    alternative = two.sided
```

Worked example

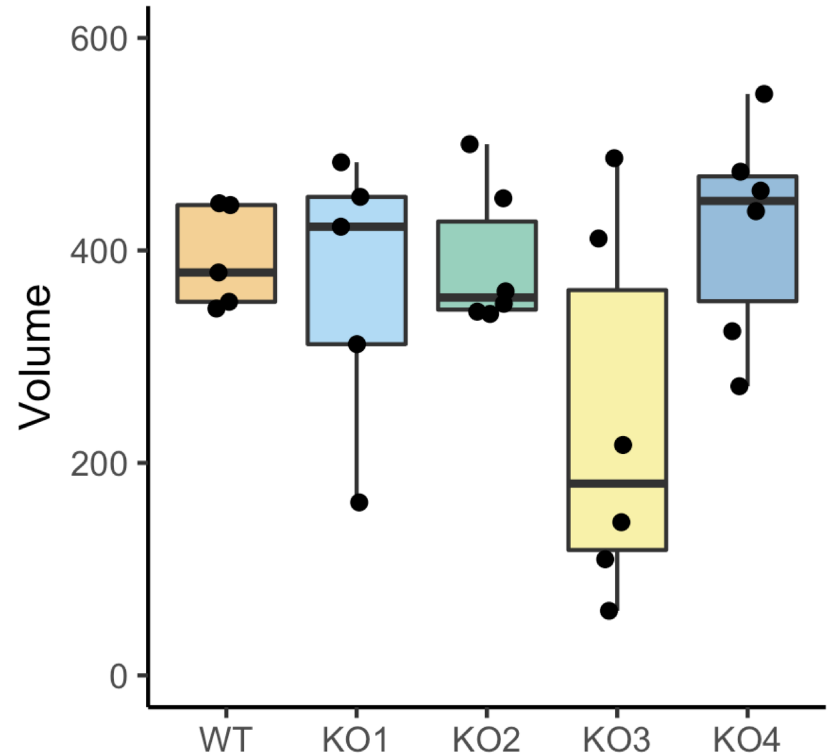
Tumour growth in mice

Pilot experiment

WT and 4 KOs mice

Observe tumour growth

Measure volume after 10 days



Power analysis

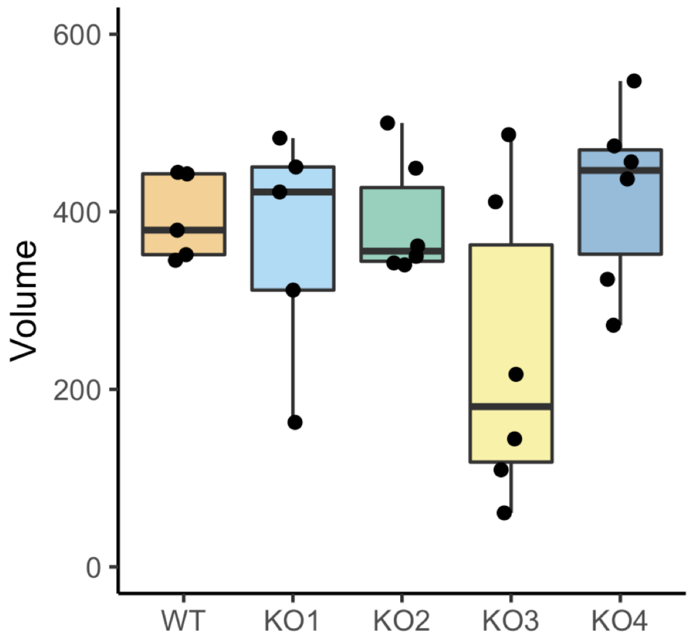
How many replicates do we need to...

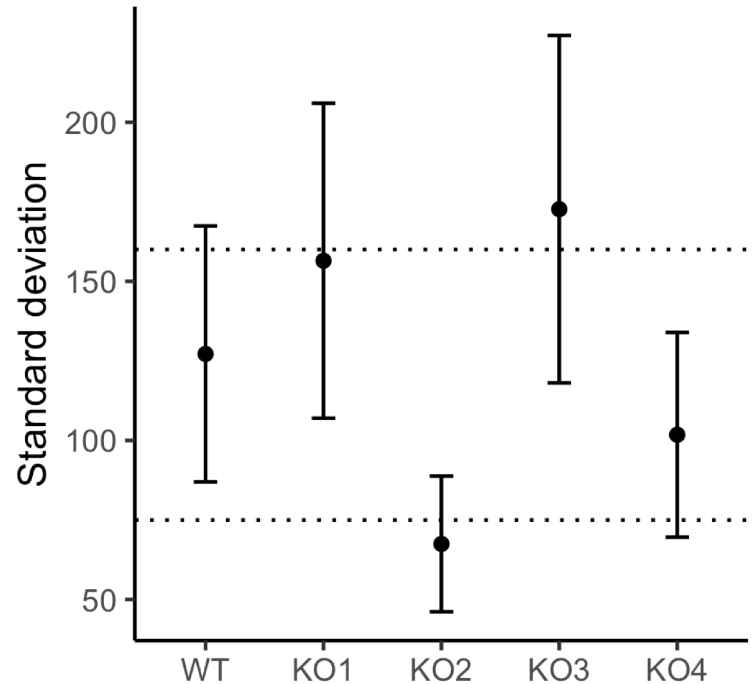
1) detect a 2-fold change between conditions? (power in t-test)

2) detect the observed effect in ANOVA? (power in ANOVA)

How many replicates to detect a 2-fold change between WT and a KO?

Estimate standard deviation





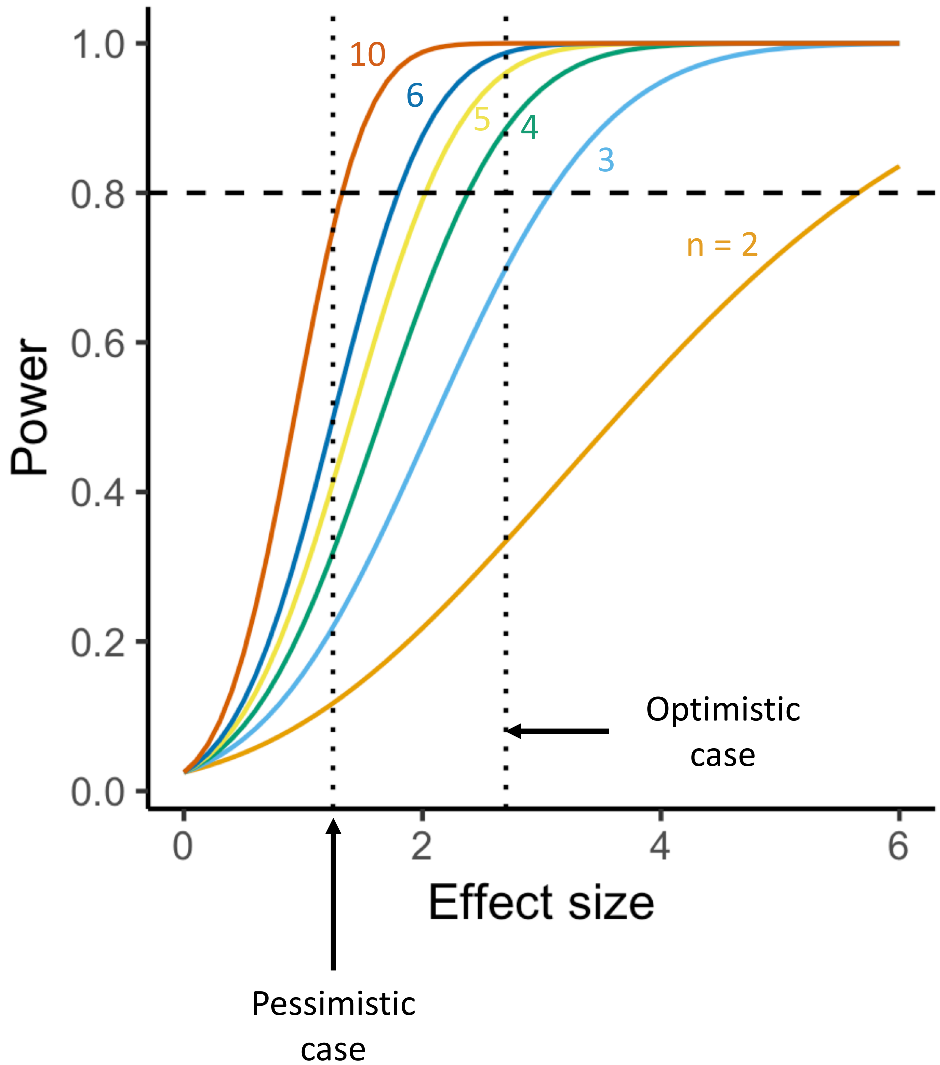
Pessimistic case
$SD = 160$

Optimistic case
$SD = 75$

Standard error of SD

$$SE_{SD} = \frac{SD}{\sqrt{2(n-1)}}$$

Two scenarios, SD_1 = 75 and SD_2 = 160



Cohen's d:

$$d_1 = \frac{\Delta M}{SD_1} = \frac{200}{75} = 2.7$$

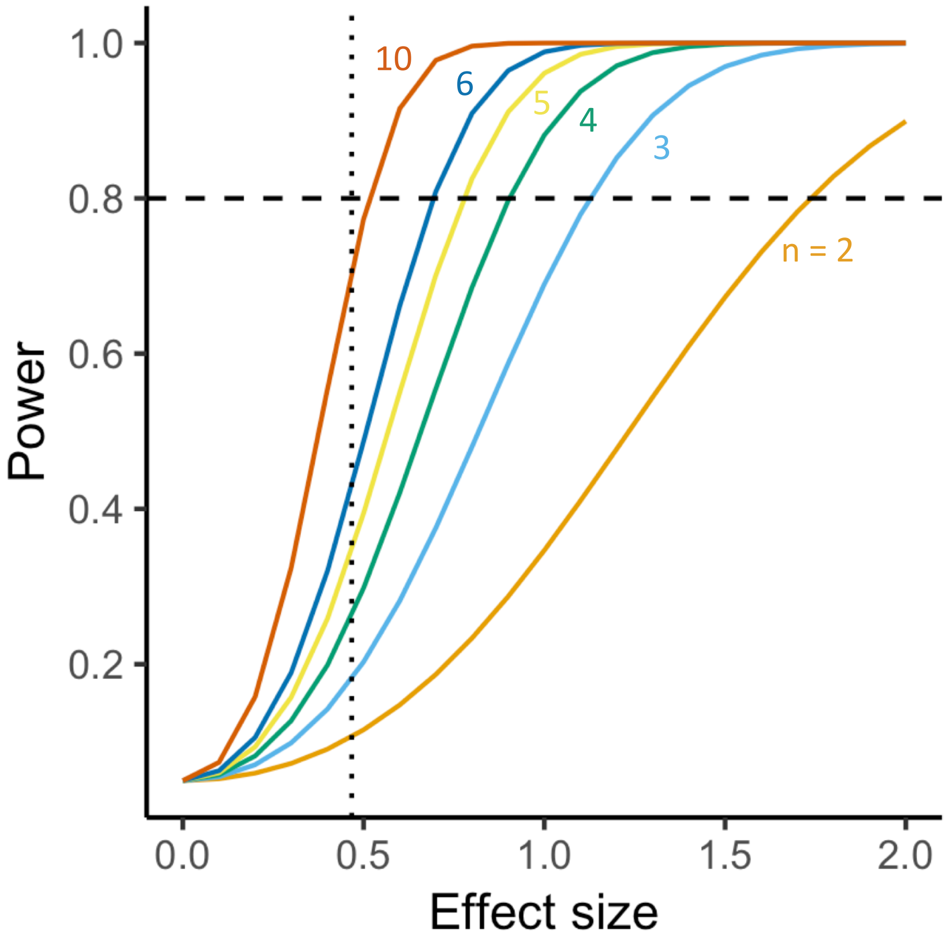
$$d_2 = \frac{\Delta M}{SD_2} = \frac{200}{160} = 1.25$$

R power calculations

```
# Optimistic case, SD = 75
> power.t.test(delta=200, sd=75, power=0.8)

     Two-sample t test power calculation

              n = 3.484297
          delta = 200
             sd = 75
      sig.level = 0.05
          power = 0.8
    alternative = two.sided

# Pessimistic case, SD = 160
> power.t.test(delta=200, sd=160, power=0.8)

     Two-sample t test power calculation

              n = 11.09423
          delta = 200
             sd = 160
      sig.level = 0.05
          power = 0.8
    alternative = two.sided
```

How many replicates to detect the observed effect in ANOVA?

ANOVA power curves



From ANOVA on our data we have $F = 2.31$

Then, we find the observed effect size:

$$f = \sqrt{\frac{F-1}{n}} = 0.47$$

How many replicates do we need?

```
> library(pwr)
> tumour <- read.table("http://is.gd/mouse_tumour", header=TRUE)
# Here n = 6 and k = 5
> tum.aov <- aov(Volume ~ Group, data=tumour)  # perform ANOVA
> F <- summary(tum.aov)[[1]]$F[1] # Extract F value
> f <- sqrt((F - 1)/6) # Effect size: Cohen's f

# What is the power of this experiment?
> pwr.anova.test(k=5, n=6, f=f)

              k = 5
              n = 6
              f = 0.4670469
      sig.level = 0.05
          power = 0.4293041
# How many replicates to get power of 0.8?
> pwr.anova.test(k=5, f=f, power=0.8)

              k = 5
              n = 11.93119
              f = 0.4670469
      sig.level = 0.05
          power = 0.8
```

Conclusions from our example

- Request power of 0.8

- To detect 2-fold change between WT and a KO in a pessimistic case we need 11 mice in each group

- To detect a change across all groups (ANOVA) we need 12 mice in each group

- We recommend an experiment with at least 12 mice in each group

Hand-outs available at

http://is.gd/statlec